Lesson Plan
Lecture Version

System of two quadratic inequalities

Objectives:

Students will:

- Graph a quadratic inequality and describe its solution set
- Solve a system of quadratic equalities graphically
- Solve a system of quadratic equalities symbolically (optional)
- Set up and solve a system of quadratic inequalities related to a real-world context

Prerequisite Knowledge

Students are able to:

- Solve a quadratic system of equations graphically
- Solve a quadratic system of equations symbolically (optional)
- Graph a quadratic function (optional)
- Find the maximum value on a graph
- Find the intersection on a graph
- Evaluate a function
- Translate written sentences to algebraic expressions

Resources

- This lesson assumes that your classroom has only one computer, from which you can lecture. For classrooms with enough computers for all your students either working individually or in small groups, see the lab version of this lesson.
- pencil, paper, and calculator
- Access to http://www.explorelearning.com/
- Copies of the worksheet for each students

Lesson Preparation

Before conducting this lesson, be sure to read through it thoroughly, and familiarize yourself with the System of two quadratic inequalities activity at ExploreLearning.com. You may want to bookmark the activity pages for your students. If you like, make copies of the worksheet for each student.

Lesson

Motivation

Business situations are often complex. Two managers are discussing the sales price of their ceramic plates. The first manager suggests that the price per plate should start at $20 and then decrease by at least 5 cents for every plate sold. This method will encourage people to buy more because the price will continually get cheaper. The other manager, however, disagrees with this method because they will be giving away their plates if the demand gets too high. Therefore, she offers an alternate sales strategy that would start selling the plates at a low price, such as $10, and then increase the price by at least 2 cents for every plate sold. The disadvantage of this strategy, however, is that the price could get so costly that consumers would not want to buy their product. After listening to the managers' suggestions, their boss proposes that they try their strategies at different locations in the country. To control unreasonable price levels, the boss tells them to stop sales once the two plans cannot support equal pricing. How much revenue should the company expect from its plate sales? How many plates should the company produce?

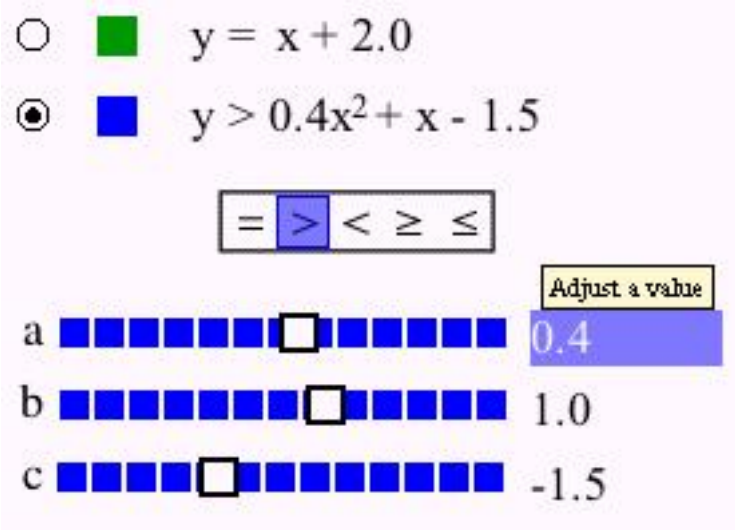
The "System of two quadratic inequalities" activity
Go to the System of two quadratic inequalities activity at ExploreLearning.com to explore a system of quadratic inequalities.

The quadratic inequality
You will start with a line and parabola graphed on the screen.

Click on the small circle (called a radio button) next to the blue function. This will allow you to modify the equation of the blue graph.

The direction of the inequality can be modified by clicking on the greater than, less than, greater than or equal to, or less than or equal to, sign.

Each value in the inequality can be modified by dragging the sliders, or by clicking to the right of the sliders and inputting a value.



Modify the equation and direction of the inequality. Have students explain why the graph is shaded, and why it is shaded in a particular direction. Ask students to distinguish the graph when the inequality has an equal sign. Have students substitute coordinates in the equation to verify the direction of the shaded region. Ask students to predict the direction of the shaded region of an inequality without using the grapher.

Multiple Inequalities
Click on the green radio button, and modify the leading coefficient 'a' so that the function is a parabola. Modify the inequalities and notice that three different types of shading will appear: blue, green, and dark blue. Make sure that students see cases where the shaded region is and is not dark blue. In the lower left-hand part of the screen, click on the box "highlight solution set." The dark blue region should change to light blue, and the green and blue regions should be removed from the graph. Ask students to explain what happens to the solution set when the two inequalities change direction. They should discuss whether the system is examining a union or intersection of the two inequalities.

Give students the graph of two quadratic equations (optional – you may want students to graph the functions by hand). Ask students to predict the graph of the region with two inequalities using these equations without using the grapher, and then check their results. For example, predict the graph of

$y \geq x^2 - 6x + 9$
$y \leq x^2 - 4x + 9$

Without using the inequality signs on the online grapher (use only the = sign), ask students to determine the equations and graph of a system of inequalities that will *not* produce an intersection region (empty set). Ask students to determine the equations and graph of a system of inequalities that has a solution with infinite area. Ask students to determine the equations and graph of a system of inequalities that has a disjoint intersection region. Ask students to determine the equations and graph of a system of inequalities that produces a symmetric intersection region. Discuss various possibilities.

Application of Quadratic Inequalities
Have students investigate the scenario at the beginning of the lesson:

Business situations are often complex. Two managers are discussing the sales price of their ceramic plates. The first manager suggests that the price per plate should start at $20 and then decrease by at least 5 cents for every plate sold (plan 1). This method will encourage people to buy more because the price will continually get cheaper. The other manager, however, disagrees with this method because they will be giving away their plates if the demand gets too high. Therefore, she offers an alternate sales strategy that would start selling the plates at a low price, such as $10, and then increase the price by at least 2 cents for every plate sold (plan 2). The disadvantage of this strategy, however, is that the price could get so costly that consumers would not want to buy their product. After listening to the managers' suggestions, their boss proposes that they try their strategies at different locations in the country. To control unreasonable price levels, the boss tells them to stop sales once the two plans cannot support

equal pricing. How much revenue should the company expect from its plate sales? How many plates should the company produce?

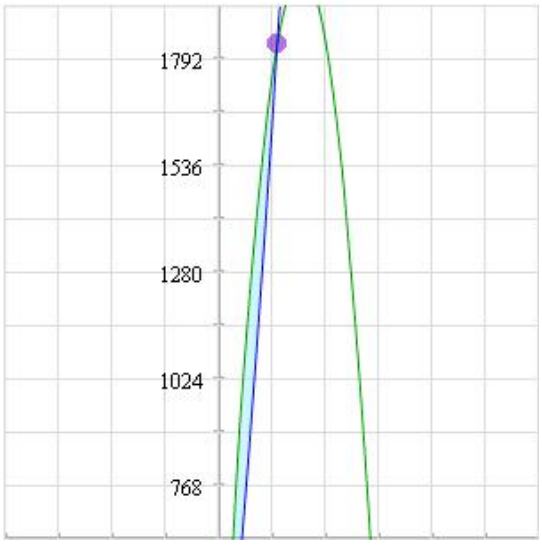
Ask students to determine strategies to solve this situation. Students need to generate two functions that relate the revenue generated by each plan compared to the number of plates sold. They also need to generate a system of inequalities that defines the region of the possible revenues based on the boss' constraints.

Students should try to generate the following equations and inequalities:
Let x = the number of plates sold
Let s_1 = the sales price for each plate under plan 1
Let s_2 = the sales price for each plate under plan 2
Let y_1 = the revenue for the plates sold under plan 1
Let y_2 = the revenue for the plates sold under plan 2

Students should determine the following inequalities about the sales price for each plate:
$s_1 \leq 20 - 0.05x$
$s_2 \geq 10 + 0.02x$

The revenue for the plates is the product of the number of plates sold and the sales price for each plate (x*s). Therefore, students should determine the following inequalities about the possible revenues for all of the plates:
$y_1 \leq 20x - 0.05x^2$
$y_2 \geq 10x + 0.02x^2$

In standard form, these inequalities are
$y_1 \leq -0.05x^2 + 20x$
$y_2 \geq 0.02x^2 + 10x$



Graph the system of inequalities. Use the zoom tools  and cross hair tools  to modify the viewing window so that you can see the intersection region on the screen.

Have students interpret the shaded region by predicting revenue given a value within its domain. For example, what is the range of expected revenues if 100 plates are sold under each plan? The cross hair tool  can be used to identify coordinates on each curve, but the students will probably get a more accurate range if they substitute 100 into each of function.

Ask students how they would use the intersection region to determine the number of plates needed and expected revenue under both plans. After the students indicate that the intersections are important, click on the "show intersection point(s)" box. Have the students verify the critical coordinates by solving the systems of equations symbolically using the quadratic formula (optional). Students should determine the revenue and number of plates needed based on these values. Note that a fractional number of plates cannot be produced. Ask students to find the average sales price for each plate.

After students determine a solution, ask students to modify the conditions of plan 2 so that the company maximizes their revenue. In addition, have students modify plan 1 if the company want a total revenue exceeding $2000 and an average revenue greater than $15 per plate.

Conclusion

The graph of a quadratic equality represents the set of points that satisfy the inequality. The shaded region will be above the quadratic if the inequality is facing to the left in standard form ($y \geq ax^2 + bx + c$), and vice versa. An equal sign in the inequality indicates that the curve is part of the solution set. A system of quadratic inequalities represents the intersection of the shaded regions for each of the inequalities. The intersection region can be used to determine a range of predicted revenue in a business situation.